SRT:

4-Vektoren, EM-Feld

Metrik, Kovariante Ableitung

LT

Ableitung

4-Vektor: $x^\mu = \begin{pmatrix} ct \\ \vec{r} \end{pmatrix}$ kontravariant, $\mu \in \{0,1,2,3\}$

$x_\mu = \begin{pmatrix} ct \\ -\vec{r} \end{pmatrix}$ kovariant

Metrik: $x_\mu = g_{\mu\nu} x^\nu \Rightarrow (g_{\mu\nu}) = \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & -\mathbb{1} \end{pmatrix}$ Minkowski-Metrik

Ableitung: $\partial_\mu = \begin{pmatrix} \frac{1}{c}\partial_t \\ \partial_i \end{pmatrix}$, $i \in \{1,2,3\}$ bzw. $\{x,y,z\}$ $\partial_i = \nabla$

$\rightarrow \partial^\mu = \begin{pmatrix} 1/c\, \partial_t \\ -\partial_i \end{pmatrix}$

EM-Feld: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $A^\mu = \begin{pmatrix} \phi \\ \vec{A} \end{pmatrix}$

$$(F_{\mu\nu}) = \begin{pmatrix} 0 & +\vec{E}^T \\ -\vec{E} & \begin{matrix} 0 & -B_z & B_y \\ B_z & 0 & -B_x \\ -B_y & B_x & 0 \end{matrix} \end{pmatrix}$$

Kovariante Ableitung: $D_\mu = \partial_\mu + \frac{iq}{\hbar c} A_\mu$

4-Norm: $s = x_\mu x^\mu$ kann sein: < 0 (raumartig), > 0 (zeitartig), $= 0$ (lichtartig)

Lorentz-Trafo: Erhält 4-Norm:

$x'^\mu = \Omega^\mu{}_\nu x^\nu$

$s = x'_\mu x'^\mu = \Omega_\mu{}^\alpha x_\alpha \Omega^\mu{}_\beta x^\beta = x_\alpha \underbrace{\Omega_\mu{}^\alpha \Omega^\mu{}_\beta}_{\stackrel{!}{=} \delta^\alpha{}_\beta} x^\beta$, da $s = x_\alpha x^\alpha$

$\delta^\alpha_\beta = \delta^\alpha{}_\beta \stackrel{!}{=} \Omega^{\nu\alpha} g_{\nu\mu} \Omega^\mu{}_\beta \quad | \cdot g_{\gamma\alpha}$

$g_{\gamma\beta} = \underbrace{\Omega^\nu{}_\alpha}_{= (\Omega_\alpha{}^\nu)^T} g_{\nu\mu} \Omega^\mu{}_\beta \Rightarrow g = \Omega^T g \Omega$

$\alpha = \beta$ wegen $\delta^\alpha{}_\beta$

$\rightarrow (\Omega^{-1\,\mu}{}_\beta)^\alpha{}_\mu = g^{\lambda\alpha} \Omega^\nu{}_\lambda g_{\nu\mu} = g^T \Omega^T g$