Beispielrechnung ideales Bosegas

$$\varepsilon = \frac{|\vec p|^2}{2m} \Rightarrow p = |\vec p| = \sqrt{2m\varepsilon}\,, \quad d\varepsilon = \frac{p}{m}\,dp \Leftrightarrow dp = \sqrt{\frac{m}{2\varepsilon}}\,d\varepsilon\,, \quad \varepsilon_{min} = 0$$

3D $$\Rightarrow d^3p = 4\pi p^2\,dp = 4\pi \cdot 2m\varepsilon \cdot \sqrt{\frac{m}{2\varepsilon}}\,d\varepsilon = 8\pi \cdot \sqrt{\frac{m^3\varepsilon}{2}}\,d\varepsilon$$

$$\sum_{\lambda \neq min} \longrightarrow (2s+1)\int_V d^3r \int_0^\infty \frac{d^3p}{(2\pi\hbar)^3} = (2s+1)V \int_0^\infty \frac{1}{\pi^2\hbar^3}\sqrt{\frac{m^3\varepsilon}{2}}\,d\varepsilon$$

$$n_B(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}-1} \equiv \frac{1}{z^{-1}e^{\beta\varepsilon}-1}$$

$$\langle N \rangle = \sum_\lambda n_B(\varepsilon_\lambda) = \underbrace{(2s+1)\frac{z}{1-z}}_{\equiv N_0} + (2s+1)V\int_0^\infty \frac{\sqrt{\varepsilon}}{z^{-1}e^{\beta\varepsilon}-1}\,\frac{1}{\pi^2\hbar^3}\sqrt{\frac{m^3}{2}}\,d\varepsilon$$

$$= N_0 + (2s+1)V\int_0^\infty \frac{\sqrt{x}}{z^{-1}e^x-1}\Big(\underbrace{\sqrt{\frac{2\pi m k_B T}{2\pi\hbar^2}}}_{\equiv\lambda_T^{-1}}\Big)^3 \frac{2}{\sqrt{\pi}}\,dx$$

$$= N_0 + (2s+1)\frac{V}{\lambda_T^3}\frac{2}{\sqrt{\pi}}\underbrace{\int_0^\infty \frac{\sqrt{x}}{z^{-1}e^x-1}\,dx}_{= \mathrm{Li}_{3/2}(z)\cdot\Gamma(\frac{3}{2})}$$

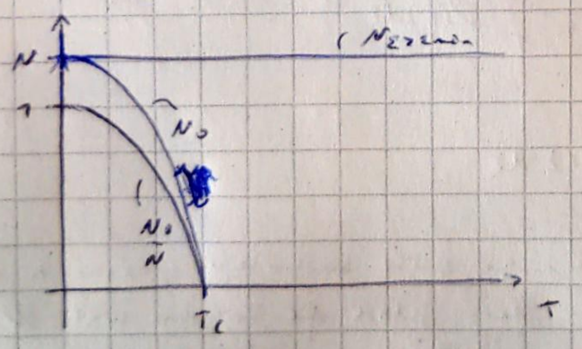
$$n = \frac{N}{V} = \frac{N_0}{V} + (2s+1)\frac{1}{\lambda_T^3}\frac{2}{\sqrt{\pi}}\mathrm{Li}_{3/2}(z)\Gamma(3/2)$$

Betrachte T_c: Bei T_c gilt $n_0 = \frac{N_0}{V} = 0$ (zumindest beim rechtsseitigen Limes $T \to T_c+$)

$$\Rightarrow n = (2s+1)\frac{2}{\sqrt{\pi}}\mathrm{Li}_{3/2}(z)\Gamma(3/2)\sqrt{\frac{mk_B}{2\pi\hbar^2}}^3 \cdot \sqrt{T_c}^3$$

Eigtl. $z = z(T)$ aber vernachlässigbar

$$\frac{N_0}{N} = \frac{n_0}{n} = \frac{n - \frac{2}{\sqrt{\pi}}\mathrm{Li}_{3/2}(z)\Gamma(3/2)\sqrt{\frac{mk_B}{2\pi\hbar^2}}\sqrt{T}^3}{n} \simeq 1 - \sqrt{\frac{T}{T_c}}^3 = 1 - \left(\frac{T}{T_c}\right)^{3/2}$$



$$N_0 = \frac{(2s+1)}{1/z - 1} \Leftrightarrow z = \frac{1}{1+\frac{2s+1}{N_0}} \overset{N_0 \gg 2s+1}{\simeq} 1 - \frac{2s+1}{N_0} = e^{\beta\mu} \overset{\mu \approx 0}{\simeq} 1 + \beta\mu \Leftrightarrow \mu \simeq -\frac{2s+1}{\beta N_0}$$

$$\Rightarrow \mu \propto \frac{-1}{\tilde{c} - T^{3/2}}$$