Wahrscheinlichkeitsdichte und -Strom ~~der~~ von Spinoren

(Kontinuitätsglg in Dirac theorie)

Es wird Glg. der Form $\partial_\mu j^\mu = 0$ gesucht

$$0 = \psi^\dagger \gamma^0 \left(i\hbar \gamma^\mu D_\mu - mc\,\mathbb{1} \right)\psi$$

$$= i\hbar \psi^\dagger \gamma^0 \gamma^\mu D_\mu \psi - mc\psi^\dagger \gamma^0 \psi$$

$$= i\hbar \psi^\dagger \gamma^0 \gamma^\mu \partial_\mu \psi + \frac{q}{c}\psi^\dagger \gamma^0 \gamma^\mu A_\mu \psi - mc\psi^\dagger \gamma^0 \psi$$

$$= i\hbar\, \partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi - i\hbar (\partial_\mu \psi^\dagger)\gamma^0 \gamma^\mu \overset{(\gamma^0)^2}{} \psi - \frac{q}{c}\psi^\dagger \overset{(\gamma^0)^2}{\gamma^0} \gamma^\mu A_\mu \psi - mc\psi^\dagger \gamma^0 \psi$$

$A_\mu = A_\mu^\dagger$ ⇒

$$= i\hbar\, \partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi$$
$$+ \psi^\dagger \left(-i\hbar \underbrace{\left(\overleftarrow{\partial}_\mu - \frac{iq}{\hbar c} A_\mu^\dagger \right)}_{=(D_\mu)^\dagger} \underbrace{\gamma^0 \gamma^\mu \gamma^0}_{=(\gamma^\mu)^\dagger} - mc \right) \gamma^0 \psi$$

$$= i\hbar\, \partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi$$
$$+ \psi^\dagger \left(-i\hbar \underbrace{D_\mu^\dagger (\gamma^\mu)^\dagger}_{=(\gamma^\mu D_\mu)^\dagger} - mc \right) \gamma^0 \psi$$

$$= i\hbar\, \partial_\mu \psi^\dagger \gamma^0 \gamma^\mu \psi + \Big[\underbrace{\left(i\hbar \gamma^\mu D_\mu - mc \right) \psi}_{=0,\ \text{Dirac-Gl}} \Big]^\dagger \gamma^0 \psi$$

→ $j^\mu \propto \psi^\dagger \gamma^0 \gamma^\mu \psi$

Definiere $j^\mu = c\, \psi^\dagger \gamma^0 \gamma^\mu \psi = \begin{pmatrix} c\,\psi^\dagger \psi \\ c\,\psi^\dagger \vec{\alpha}\, \psi \end{pmatrix} = \begin{pmatrix} \rho c \\ \vec{j} \end{pmatrix}$

→ Die Wahrscheinlichkeitsdichte ist

$$\rho = \psi^\dagger \psi = |\psi_0|^2 + |\psi_1|^2 + |\psi_2|^2 + |\psi_3|^2$$

Die Wsk-Stromdichte ist $\vec{j} = c\, \psi^\dagger \vec{\alpha}\, \psi = c \begin{pmatrix} \psi^\dagger \begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix} \psi \\ \psi^\dagger \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix} \psi \\ \psi^\dagger \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \psi \end{pmatrix}$