Ideales Bosegas für hohe Temp.

$$\varepsilon = \frac{p^2}{2m}, \quad \varepsilon_{min} = 0, \quad dp = \sqrt{\frac{m}{2\varepsilon}}\, d\varepsilon, \quad d^3p = 4\pi p^2 dp \quad \nu(\varepsilon) = \frac{m^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3}\sqrt{\varepsilon} \propto \sqrt{\varepsilon}$$

$$\Omega = +k_B T \sum_\lambda \ln\left(1 - e^{-\beta\varepsilon_\lambda} z\right)$$

große

$$\frac{1}{\lambda_T} = \sqrt{\frac{m}{2\pi\beta\hbar^2}} \qquad \partial_T\left(\frac{1}{\lambda_T}\right)^3 = \frac{3}{2}\sqrt{\frac{m k_B}{2\pi\hbar^2}}^{\,3} \sqrt{T}$$

$$= (2s+1) k_B T \ln(1-z) + k_B T (2s+1) V \int \frac{m^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\sqrt{\varepsilon}\ln\left(1 - z e^{-\beta\varepsilon}\right) d\varepsilon = (2s+1) k_B T \ln(1-z) + (2s+1) V \frac{2 k_B T}{\sqrt{\pi}\,\lambda_T^3}\underbrace{\int \sqrt{x}\ln\left(1 - z e^{-x}\right) dx}_{\simeq -\frac{\sqrt{\pi}}{2} Li_{5/2}(z)}$$

$$\simeq (2s+1)\Big[\underbrace{k_B T \ln(1-z)}_{} + k_B T \frac{V}{\lambda_T^3} Li_{5/2}(z)\Big]$$

Verschwindet für $T > T_c$

$\rightarrow$ μ mangeln $\hat{=}$ dominieren

$$\simeq (2s+1) k_B T \frac{V}{\lambda_T^3} Li_{5/2}(z) \qquad , \quad Li_s(z) = \sum_{n=1}^\infty \frac{z^n}{n^s} \rightarrow z\, Li_s'(z) = z \sum_{n=1}^\infty \frac{n z^{n-1}}{n^s} = \sum_{n=1}^\infty \frac{z^n}{n^{s-1}} = Li_{s-1}(z)$$

$$n = \frac{N}{V} = -\partial_\mu \Omega\big|_{V,T} = (2s+1) \cancel{k_B T} \frac{V}{\lambda_T^3} Li_{3/2}(z) \cancel{\beta}$$

bei T_c gilt $z = 1$

Betrachte n - fixiert :

$$n = (2s+1)\frac{V}{\lambda_T^3} Li_{3/2}(z) \overset{!}{=} (2s+1)\frac{V}{\lambda_{T_c}^3}\underbrace{Li_{3/2}(1)}_{= \text{const.} = \zeta(3/2)}$$

$$\Leftrightarrow \quad Li_{3/2}(z) = Li_{3/2}(1)\left(\frac{\lambda_T}{\lambda_{T_c}}\right)^3 = Li_{3/2}(1)\left(\frac{T_c}{T}\right)^{3/2}$$

Außerdem $\quad Li_{3/2}(1-y) \simeq 1 - 2\sqrt{\pi}\sqrt{y}\,,$

und $\quad \exp^{-y} \simeq 1 - y$

$$\overset{y = -\beta\mu > 0}{\Longrightarrow} \quad 1 - 2\sqrt{\pi}\sqrt{\beta\mu} = \underbrace{Li_{3/2}(1)}_{\equiv \zeta}\left(\frac{T_c}{T}\right)^{3/2}$$

$$\mu = -\frac{1 - 2\zeta\left(\frac{T_c}{T}\right)^{3/2} + \zeta^2\left(\frac{T_c}{T}\right)^3}{4\pi} \cdot k_B T$$

$$= k_B\left(-\frac{T}{4\pi} + \frac{\zeta\, T_c^{3/2}}{2\pi\sqrt{T}} - \frac{\zeta^2 T_c^3}{4\pi T^2}\right)$$

$$-\frac{\mu}{k_B T^2} \quad z\, Li'_{3/2}(z) = Li_{1/2}(z)$$

Druck: $\quad P = -\frac{\Omega}{V}$

$$P = -(2s+1)\left[\frac{k_B T\ln(1-z)}{V} + \frac{k_B T}{\lambda_T^3} Li_{5/2}(z)\right]$$

$\rightarrow 0$ im TD Limes $\Rightarrow$ Kondensat trägt nicht zum Druck bei!

Kritischer Druck bei $T = T_c \rightarrow z = 1 \rightarrow P_c = -\frac{k_B T_c}{\lambda_{T_c}^3} Li_{5/2}(1)$

Entropie:

$$S = -\frac{\partial\Omega}{\partial T}\Big|_{V,\mu} = -(2s+1)\left[k_B\ln(1-z) - \frac{\mu}{T^2}\frac{k_B T}{\frac{1}{z}-1} + \frac{k_B V}{\lambda_T^3} Li_{5/2}(z) + \frac{3}{2}\frac{k_B V}{\lambda_T^3} Li_{5/2}(z) - \frac{k_B T V}{\lambda_T^3} Li_{3/2}(z) \cdot \frac{\mu}{T^2 k_B}\right]$$

$$= (2s+1)\left[-k_B\ln(1-z) + \frac{\mu}{T}\left[\frac{1}{z^{-1}-1} + \underbrace{\frac{V}{\lambda_T^3} Li_{3/2}(z)}_{= \frac{V}{\lambda_{T_c}^3} Li_{3/2}(1) = \text{const.}}\right] - \frac{5}{2}\frac{k_B V}{\lambda_T^3} Li_{5/2}(z)\right]$$

Da μ - fixiert gilt für $T \to 0$ $(T \ll T_c)$: $z = e^{\beta\mu} \to 0$ (da $\mu < 0$) für die Beiträge aus dem Kontinuum.

Für die Beiträge aus dem Integral gilt $z \simeq 1$

$$\Rightarrow \quad S \to 0 \quad (T \to 0) \quad ✔, \text{ 3. Hauptsatz}$$